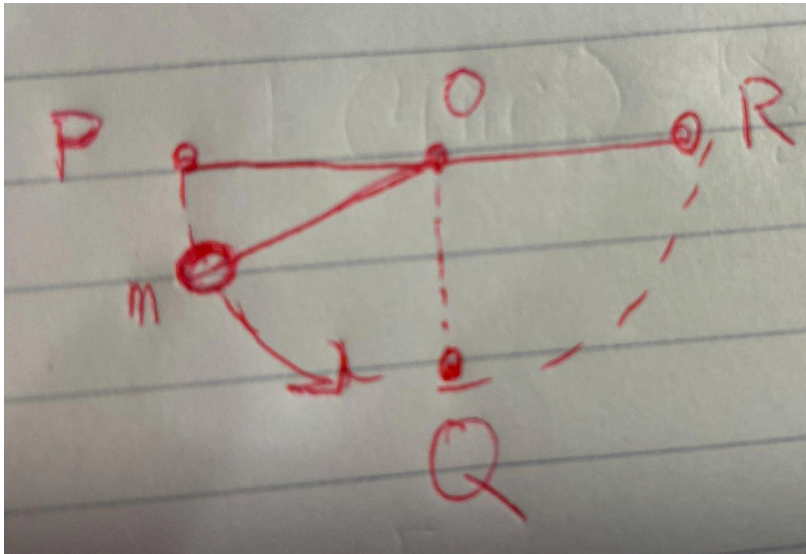


Background: Centripetal motion (circular motion) refers to objects moving in circular paths. Therefore, this unit is mainly about finding the force keeping an object moving in a circle or its acceleration while in circular motion. Major topics in this sector include centripetal force, centripetal acceleration, angular velocity, and tangential velocity.

<u>Key Points and Phrases</u>	<u>Formulas Necessary</u>	<u>More Formulas Necessary</u>
<ol style="list-style-type: none"> Centripetal force is the force directed toward the center of a circular path that keeps the object moving in a circle. Centripetal acceleration is the acceleration directed towards the center of a circular path. It is perpendicular to the velocity of the object. Angular velocity is the rate of change of angular displacement. Tangential velocity is the velocity of the object along the tangent of its circular path If the object is moving in a uniform circular motion, the magnitude of the velocity must be the constant The period (T) is the inverse of the frequency (f) 	$\omega = v/r$ <p>This equation shows the relationship between angular and tangential velocity.</p> <ul style="list-style-type: none"> ω is the magnitude of the angular velocity v is the magnitude of the tangential velocity r is the radius of the circular path $\omega = \theta/t$ <p>This equation shows the angular velocity of the object</p> <ul style="list-style-type: none"> ω is the magnitude of the angular velocity θ is the angular displacement T is the time taken 	$F_c = (mv^2)/r$ <p>This is the equation for centripetal force.</p> <ul style="list-style-type: none"> F is the magnitude of the centripetal force m is the mass of the object v is the velocity of the object r is the radius of the circular path $a_c = v^2/r$ <p>This is the equation for centripetal acceleration.</p> <ul style="list-style-type: none"> a is the magnitude of the centripetal acceleration v is the velocity of the object r is the radius of the circular path

Practice Problems

1. [Easy] A ball of mass m is attached to the end of a string of length Q as shown above. The ball is released from rest from position P where the string is horizontal. It swings through position Q where the string is vertical, and then to position R where the string is again horizontal. What are the directions of the acceleration vectors of the ball at positions Q and R?

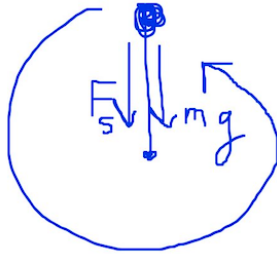


(It's a pendulum!)

Solution:

At position Q, the acceleration vector is upwards because the centripetal acceleration always points toward the center of the circle (in this case upwards). At position R, the acceleration is downward. The ball stops moving as it hits position R because it is at the peak of its swing. Therefore, the tension is equal to zero and the only force acting on the ball is gravity pulling the ball downward.

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2. [Medium] One end of a string is fixed. An object attached to the other end moves on a horizontal plane with uniform circular motion of radius R and frequency f . The tension in the string is F . If both the radius and frequency are doubled, the tension is what?



Answer:

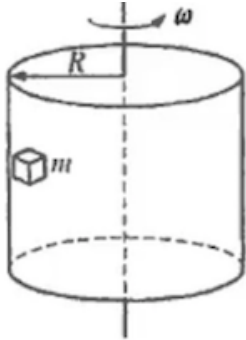
$$\begin{aligned} \omega &= 2\pi f = \frac{v}{R} \\ v &= 2\pi f R \\ F &= m \frac{v^2}{R} \\ F &= m (2\pi f)^2 R \\ F &= 4m\pi^2 f^2 R \end{aligned}$$

This is the equation for the tension before the radius and frequency are doubled.

$$\begin{aligned} \omega &= 2\pi f = \frac{v}{R} \\ v &= 2\pi f R \\ F &= m \frac{v^2}{R} \\ F &= m (2\pi f)^2 R \\ F &= 4m\pi^2 f^2 R \\ \\ F &= m (2\pi 2f)^2 2R \\ &= 4m\pi^2 f^2 2R \\ &= 32m\pi^2 f^2 R \end{aligned}$$

This is the equation for the tension after the radius and frequency are doubled. They are basically the same except that the coefficient for the first equation is 4, while the second is 32. This means that the tension after the radius and frequency are doubled is 8 times what it was before. This means that the answer is 8F.

3. [Harder] A block of mass m is placed against the inner wall of a hollow cylinder of radius R that rotates about a vertical axis with a constant angular velocity ω as shown in the diagram below. In order for friction to prevent the mass from sliding down the wall, the coefficient of static friction must be at a minimum what? Use the R , ω , and any constants.



Solution:

Handwritten solution on lined paper:

Free-body diagram of the block showing forces: Friction (up), Normal force N (right), and gravity mg (down).

N is the centripetal force and it is pointing towards the center of the cylinder.
 $N = m \frac{v^2}{R}$

$F_{\text{friction}} = mg$
 $N \mu_s = mg$
 $\mu_s = \frac{mg}{N}$

$\mu_s = \frac{mgR}{mv^2}$
 $\mu_s = \frac{gR}{v^2}$

$\mu_s = \frac{g}{\omega^2 R}$
 $\omega = \frac{v}{R}$ $v = \omega R$

Since the block is not moving up or down, we can set the force of friction equal to the force of gravity. The force of friction is equal to the normal force (N) times the static coefficient of friction. The normal force is the centripetal force as it points towards the center of the cylinder meaning it is equal to mass times the centripetal acceleration. Substituting in this value for N , we can find the static coefficient of friction value in terms of g , ω , and R . The static coefficient of friction is equal to $g/(\omega^2 R)$ at minimum.